SCATTERED THOUGHTS ABOUT MATHEMATICS AND EDUCATION

JAVIER A. MORENO

1. Mathematics is a hidden sense

We feel mathematics. Our mind is designed to perceive and process numbers, patterns and structures almost the same way it processes sights or sounds. It is natural to us. We count stones (and then cattle, and then steps on the moon); we measure change; we detect and exploit orders and symmetries; we classify and organize information. This perceptual channel, however, needs to be awaken and trained, otherwise it barely evolves.

Learning mathematics is more about the refinement and control of an intuition (or a set of intuitions) than about the accumulation of facts and mechanical techniques. In order to properly assimilate mathematics, so that you can appreciate its beauty and fully understand its limits, you need to experience it as if it was another universe. That is what I try to offer to my students when I teach. I try to guide them so they can perceive by themselves these objects and start playing with them. Achieving this is hard, but it is wonderful when it happens. Serious multiverse travelling is never easy at first.

2. Mathematics is (also) a language

We need a language to tell the story of what we have seen. The abstract nature of the mathematical experience has required us to come up with formal languages designed to help us share and also study the realities and creations accessible through this extra sense. These are very strict languages equipped with their own syntax and grammar. Sometimes they feel like symbolic games but they act as (perceptual) portals: through them we are able to explore, experiment and manipulate the structures that inhabit the different mathematical universes. In a way, practicing the languages with proper supervision enhances our intuition and vice versa. They also serve as a bridge between the natural world and mathematics. In class, I make constant emphasis on the importance of communicating and recording ideas with clarity and precision, and I show my students how these languages facilitate the task.

3. PROBLEMS COME FIRST

We need motivations, reasons to go there. Traditional mathematical education provides the student with solutions to problems they have never faced before. They do not know why these problems are interesting or challenging. The problems arise almost as a sad justification for the time spent discussing the technique. As an instructor at the University of Illinois, I had the opportunity to try the opposite approach. I feel this approach is not only more natural because it is similar to the way mathematics has been constructed/discovered, but also more effective and personally fulfilling.

Whenever the setting permits, at the beginning of each class I introduce carefully chosen problems–sometimes with historical roots, sometimes out of real situations, sometimes purely recreational–and then I let the students play in small groups with the problems so they can assess on their own the difficulty and also attempt naïve solutions. I want the students to feel the urge for solving the problems and to care about the problems, before I guide them through questions towards a solution that illustrates a more general concept or technique. Then, I encourage the students to complement what they have learned by exploring new problems that evidence the range of applications of the concept as well as its limitations. My task from then on is to monitor their progress and suggest questions and variants that provide insight, disrupt mechanical approaches, and test their freshly acquired intuition.

I believe good and careful problem design has been unjustifiably overlooked by traditional mathematical education. The generic (empty) problem is nowadays the norm. This should change.

4. SOCIAL INTERACTION ENRICHES LEARNING

Mathematics is a social activity. It has been created and developed by people who have discussed problems and puzzles with other people. It was, and still is, a social game. That is the way it grew and evolved. Given this, I find it odd that in traditional undergraduate mathematics classes, with the immense lecture halls and industrialized marking, everything is designed to reduce social interaction and collaboration to its minimum.

In my ideal learning environment, which I had the pleasure of achieving a few times at the University of Illinois, students know and trust each other and collaborate. I also know them and they trust me. I am neither enemy nor obstacle. I know (most of) their names and have also an idea of their interests, backgrounds and motivations. They are better students and I am a better instructor when we are not a bunch of strangers sharing a room thrice a week. That is the reason I tell them about myself and ask them about themselves. Even when teaching in a large lecture hall, I work hard to generate a sense of community in class, sometimes with more success than others. I also encourage students to use my office hours to discuss problems one-on-one. I try to be as approachable as possible. When students loosen up, it is easier to detect their difficulties and strengths and, in turn, I am better able to focus my explanations and questions according to each case. I can also rely on them to help fellow classmates. The exchanges facilitate and enliven the process.

5. Experimental mathematics is possible

There is an unresolved tension between mathematical education and the use of computers. Despite the fact that most people deal with mathematics using computers, active use of computers has not been included as part of the standard mathematical curriculum. We artificially disconnect two activities that will be inseparable when it comes to actually apply what has been learned.

As I see it, computers can be marvelous tools to experience and discover mathematics. At the University of Illinois, I had the opportunity to teach a couple of courses within the Calculus and Mathematica program. These courses were conducted in computer labs where students would review the basics of calculus by executing and solving a series of Mathematica-based assignments. Ideas were graphically illustrated and the students had the chance of playing with techniques and concepts in an easy, interactive way. This approach allowed me to focus class discussions on the deep conceptual machinery behind the superficial calculations and the physical, historical and philosophical ideas that motivated the creation of these tools. I found that the students in this type of course were better at visualizing mathematical concepts, which gave them an advantage over calculus students from more traditional sections. Through computers and programming, students can really experiment with the concepts they are learning and get a deeper understanding of the way they work. Programming skills also provide students with a practical way of formalizing and testing arguments and procedures.

6. Mathematical tools are essentially constrained

We use mathematics, but we can also abuse mathematics. Mathematical tools, although reliable and precise, are extremely limited in their range of applications and should be used with extreme care. Usually, natural phenomena must be drastically simplified in order to fit the requirements. It may be also necessary to add artificial assumptions that restrict the scope of the results even more. A mathematical statement never truly addresses reality; it addresses mathematics and mathematical models. The model and the object to be modelled are certainly related but should never be confused. Unfortunately, there is such a thing as irresponsible use of mathematics and this type of confusion is sometimes intentionally promoted. The recent financial crisis shows how serious this can be. I believe mathematical education should prepare the students to be aware of these limitations–not everything is quantifiable and not every quantification is valid or really useful–and also to detect misuse or abuse of mathematical techniques.

7. Higher education should provide more than titles and connections

To finish, let me share with you a story: I am teaching two sections of differential calculus for sciences this term. Each section has about ninety students. My contact with the students is very limited. The course is designed to be a filter. They are not in class to learn but to be classified by their marks. Some will pass and some will fail (in more or less fixed percentages) and it does not matter what they were supposed to learn. The content of the course does not take into consideration their interests. These are courses designed for nobody. There is no motivation for the students to take courses such as these, except to fulfill the program requirement. During office hours, I am often visited by students who are frustrated and stressed. There is too much pressure on them compared with the amount of advice they receive. I try to help but it is never enough. Some students simply do not know how to study. Some are overloaded, taking seven first-year courses at the same time. Some (most!) simply do not know what are they doing with their lives. They are on their own and they are lost. The university, as an institution, does not care about them as young individuals with aspirations and problems. Education, actually, is not its main priority.

I believe students deserve better than that.

Undergraduate education should emphasize close personal guidance and mentoring, which should serve as the basis from which any other objective is attained. Courses should offer the students a way to explore their interests and potentials and also to challenge their beliefs. Courses and professors should be sources of inspiration. They should turn the students into critical thinkers, avid readers, skilled communicators and healthy skeptics, intellectually rounded individuals full of curiosity and a global perspective, and enthusiastic about knowledge and culture in all its complexity. Evaluation should be a tool to keep track of their evolution and provide them with proper advice and support.

That is the kind of education I would like to offer.